Effects of relativistic electrons on the calculated collective Thomson scattered spectra

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The effects of relativistic electrons on the spectral density function $S(\mathbf{k},\omega)$ are evaluated in the collective Thomson scattering regime for ion diagnostics. An electromagnetic code that calculates the electron density fluctuation term of $S(\mathbf{k},\omega)$ has been modified by treating the electrons with a relativistic Maxwellian susceptibility tensor. Results are presented for calculated scattered spectra representative of the International Thermonuclear Engineering Reactor plasma parameters. Minor effects of relativistic electrons can be seen in the $S(\mathbf{k},\omega)$ spectra only in scattering regimes where plasma resonances are present. Relativistic effects in $S(\mathbf{k},\omega)$ are negligible in other scattering regimes where plasma resonances are not observable in the spectra. Relativistic effects are more dramatic at higher fluctuation frequencies (~ 100 GHz), where the electron Bernstein resonances and electron plasma frequency resonances can be observed. Significant broadening of Bernstein resonance features toward lower frequencies and overlapping of resonance harmonic features at high electron temperatures were noted in the calculated scattered spectra.

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I. INTRODUCTION

Scattering of electromagnetic waves by a plasma offers a potentially powerful diagnostic tool to make spatially resolved measurements of fusion product α particle velocity distributions and densities in the core of fusion burning plasma devices. Collective Thomson scattering is a scattering process where the electromagnetic waves are most strongly scattered by collective electron fluctuations with scale lengths greater than the Debye shielding electrons moving with the ions in a plasma. Such collective electron fluctuations are representative of individual ion motions and plasma waves. The condition for collective Thomson scattering is given by the Salpeter parameter $\alpha \equiv 1/(|\mathbf{k}|\lambda_D) > 1$, where $|\mathbf{k}| = |\mathbf{k}_s - \mathbf{k}_0|$ is the fluctuation wave number as determined by the incident and scattered wave vectors and λ_D is the Debye length . Collective Thomson scattering is considered a leading technology for experimental studies of α particle physics in future large D-T burning experiments such as International Thermonuclear Engineering Reactor (ITER). In addition, experiments are currently underway using gyrotron sources to test this diagnostic on a Tokamak Fusion Test Reacter (TFTR) and a Joint European Torus (JET) [1,2].

The collective Thomson scattering cross section can be broadly divided into two parts, the spectral density function $S(\mathbf{k},\omega)$, where $\omega = \omega_s - \omega_0$, which depends on the velocity distributions and densities of the plasma species, and the geometrical form factor which determines the coupling between the incident and scattered beams inside a plasma. Both these parts also depend on the plasma dielectric properties which can significantly affect the scattering cross section in the vicinity of plasma resonances and cutoffs. The issue of relativistic effects on the calculation of the geometrical form factor has been addressed by Bindslev [3]. However, all collective Thomson scattering codes to date have assumed nonrelativistic electron in the modeling of $S(\mathbf{k},\omega)$.

It is well known that electrons in fusion burning plasma are mildly relativistic and that it is important to include relativistic effects when using Thomson scattering for electron temperature measurements [4]. In this paper we take a first look at relaxing the nonrelativistic assumption in an $S(\mathbf{k},\omega)$ code for calculations in the collective Thomson scattering regime for ion measurements. We modify an electromagnetic code which calculates the electron density fluctuation terms of $S(\mathbf{k},\omega)$ by treating the electrons with a relativistic Maxwellian susceptibility tensor. Results are presented for calculated scattered spectra representative of ITER plasma parameters.

II. SPECTRAL DENSITY FUNCTION: $S(\mathbf{k},\omega)$

Noting that the fluctuation current spectrum has contributions from the electrons and all the ion species in the plasma, the spectral density function can be expressed as a sum of separate contributions from the electron feature and the ion features,

$$S(\mathbf{k},\omega) = S_e(\mathbf{k},\omega) + \sum_j S_j(\mathbf{k},\omega) \ , \tag{1}$$

where if the velocity distributions of the electron and ion species are in equilibrium, then [5]

$$S_e(\mathbf{k},\omega) = \left[\frac{\mathbf{k}}{\omega} + \frac{\mathbf{k}}{\omega} \cdot \bar{\bar{\chi}}_e \cdot \bar{\bar{\mathbf{K}}}^{-1} \right] \times \left[\frac{\mathbf{k}}{\omega} + \frac{\mathbf{k}}{\omega} \cdot \bar{\bar{\chi}}_e \cdot \bar{\bar{\mathbf{K}}}^{-1} \right]^* : \left[\frac{\bar{\bar{\chi}}_e - \bar{\bar{\chi}}_e^{\,t}}{2i} \right] \frac{v_e^2 \omega}{\omega_{pe}^2} \ , \tag{2}$$

$$S_j(\mathbf{k},\omega)=\left[\frac{\mathbf{k}}{\omega}\cdot\bar{\bar{\chi}}_e\cdot\bar{\bar{\mathbf{K}}}^{-1}\right]\times\left[\frac{\mathbf{k}}{\omega}\cdot\bar{\bar{\chi}}_e\cdot\bar{\bar{\mathbf{K}}}^{-1}\right]^*:\left[\frac{\bar{\bar{\chi}}_j-\bar{\bar{\chi}}_j^t}{2i}\right]\frac{v_j^2\omega}{\omega_{pj}^2}\left[\frac{q_j^2 n_j}{n_e}\right]. \quad (3)$$

For species j, ω_{pj} is the plasma frequency, q_j is the change density, n_j is the density, v_j is the averaged velocity, $\bar{\bar{\mathbf{K}}}^{-1}$ is the inverse of the plasma dispersion tensor, and $\bar{\bar{\chi}}_e$ and $\bar{\bar{\chi}}_j$ are the susceptibility tensors of the electron and ion species, respectively.

III. RELATIVISTIC SUSCEPTIBILITY TENSOR

In the collective Thomson scattering regime, the electron spectral feature is generally much weaker than the ion features. Therefore, the spectral density function is dominated by the contributions from the $S_j(\mathbf{k},\omega)$. However the dispersion tensor $\bar{\bar{\mathbf{K}}}$, which is an integral part to both $S_j(\mathbf{k},\omega)$ and $S_e(\mathbf{k},\omega)$, is determined by all plasma species including electrons. Therefore, even in the collective scattering regime where the contribution from $S_e(\mathbf{k},\omega)$ is minor, relativistic electrons still can affect the spectral density function. Especially in situations where a plasma resonance is present in the scattered spectrum, relativistic effects are quite evident. A computer code SMAG has been developed which models the relativistic electron susceptibility tensor with a mildly relativistic model as derived by Shkarofsky [6]. The nonrelativistic susceptibility tensor is used to model the ion species; since the equations are well known, the reader is referred to the literature for details [7].

For the case of a slightly relativistic plasma, asymptotic expansions can be made to simplify the expression of the relativistic susceptibility tensor [8,9]. This approximation enables the integral to be expressed in terms of the plasma dispersion function just as in the nonrelativistic case [10,11]. In the mildly relativistic plasma case, the plasma susceptibility tensor can be expressed in terms of the Shkarofsky function,

$$F_q(\phi,\psi)=-i\exp(-\psi^2)\times\int_0^\infty dt(1-it)^{-q}\exp[-i\phi^2 t+\psi^2/(1-it)]\ , \quad (4)$$

where $\phi=k_\parallel c^2/(\omega v_j),\psi^2=\phi^2+\eta\zeta,\zeta=(\omega-N\Omega_j)/\omega,\eta=2c^2/v_j^2,N$ is an integer value, and Ω_j is the jth species cyclotron frequency.

Using the defined F_q function, the susceptibility tensor for electrons can now be expressed as

$$\bar{\bar{\chi}}_e=\begin{bmatrix}\chi_{11} & \chi_{12} & \chi_{13}\\ -\chi_{12} & \chi_{22} & \chi_{23}\\ \chi_{13} & -\chi_{23} & \chi_{33}\end{bmatrix}, \quad (5)$$

where the individual elements are

$$\chi_{11}=-\frac{\eta\omega_{pj}^2}{\omega^2}\sum_{N=-\infty}^{\infty}\sum_{p=0}^{\infty}a_{pn}n^2\lambda^{p+n-1}F_{p+n+3/2}\ , \quad (6)$$

$$\chi_{12}=\frac{i\eta\omega_{pj}^2}{\omega^2}\sum_{N=-\infty}^{\infty}\sum_{p=0}^{\infty}a_{pn}N(p+n)\lambda^{p+n-1}\times F_{p+n+3/2}\ , \quad (7)$$

$$\chi_{13}\approx-\frac{\eta\omega_{pj}^2c^2k_\perp k_\parallel}{\omega^3\Omega_j}\sum_{N=-\infty}^{\infty}\sum_{p=0}^{\infty}a_{pn}N\lambda^{p+n-1}F'_{p+n+5/2}\ , \quad (8)$$

$$\chi_{22}=-\frac{\eta\omega_{pj}^2}{\omega^2}\sum_{N=-\infty}^{\infty}\sum_{p=0}^{\infty}a_{pn}\left[(p+n)^2-\frac{p(p+2n)}{2n+2p-1}\right]\times\lambda^{p+n-1}F_{p+n+3/2}\ , \quad (9)$$

$$\chi_{23}\approx-\frac{i\eta\omega_{pj}^2c^2k_\perp k_\parallel}{\omega^3\Omega_j}\sum_{N=-\infty}^{\infty}\sum_{p=0}^{\infty}a_{pn}(p+n)\times\lambda^{p+n-1}F'_{p+n+5/2}\ , \quad (10)$$

$$\chi_{33}=-\frac{\eta\omega_{pj}^2}{\omega^2}\times\sum_{N=-\infty}^{\infty}\sum_{p=0}^{\infty}a_{pn}\lambda^{p+n}[F_{p+n+5/2}+2\psi^2F''_{p+n+7/2}]\ , \quad (11)$$

where $\lambda=(k_\perp v_e/\Omega_e)^2,n=|N|$, and the coefficient a_{pn} is given by

$$a_{pn}=\frac{(-1)^p(n+p-1/2)!}{[p!(n+p/2)!(n+p/2-1/2)!2^n]}\ . \quad (12)$$

For scattering orientation where $k_\parallel\approx 0$, the number of summations needed to calculate the electron susceptibility is small. For calculating the relativistic electron susceptibility in the collective scattering regime, sums of the N terms in the -2 to $+2$ range are sufficient because only the fundamental electron cyclotron harmonic ($N=0$) term is really significant. Moreover, because of the rapid convergence of the infinite series of p, only the $p=0$ term needs to be calculated.

IV. EFFECTS OF RELATIVISTIC ELECTRONS ON $S(\mathbf{k},\omega)$

Effects of relativistic electron correction of $S(\mathbf{k},\omega)$ are minor in the collective scattering regime. Small downward shifting of the lower hybrid (LH) resonance frequency is observed due to the relativistic increase in the mass of electrons. The main effect can be seen only near resonances where the relativistic electrons affect the plasma dispersion. Away from any resonances, relativistic effects are negligible.

Figure 1 shows the effects of relativistic electrons on the LH resonance feature for calculated scattered spectra representative of ITER plasma parameters using a 152 μm wavelength laser for this diagnostic. Slight downward shifting of the resonance frequencies can be ob-

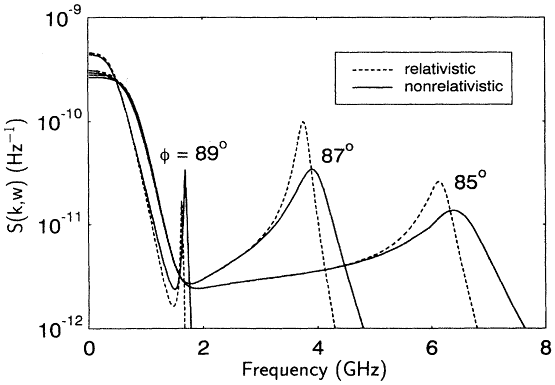

FIG. 1. Comparison of relativistic and nonrelativistic model spectra in ITER plasma in the low shifted frequency range (~ 10 GHz). The figure shows values of $S(k,\omega)$ in Hz^{-1} vs frequency in GHz. Note that resonance peaks are shifted to lower frequencies for the relativistic cases. Source wavelength is 152 μm; **B** is 5 T; electron density is 1×10^{14} cm^{-3}; electron, deuterium, and tritium temperatures are at 20 keV; 1% α particles are modeled with the slowing down velocity distribution; and the Salpeter parameter is 5.1. θ is at 4°, which is the angle between $\mathbf{k}_s$ and $\mathbf{k}_0$, and ϕ is the angle between **k** and **B**. Scattering and plasma parameters are the same for all figures except where otherwise noted.

served. Differences between relativistic and nonrelativistic models decrease as the **k** to **B** angle ϕ approaches 90°. Some slight narrowing of the resonance features are also noticeable in the plots calculated with the relativistic model. For scattering orientations where the LH resonance feature is not present ($\phi\approx0°$), relativistic effects are negligible.

Effects of relativistic electrons are more dramatic at higher fluctuation frequencies (~ 100 GHz) where the electron Bernstein resonances and electron plasma frequency resonances can be observed. Figure 2 shows calculated scattered spectra for frequencies out to Ω_e. The dispersion describing the first harmonic Bernstein resonances, within the electrostatic limit and $k_\perp\rho_e \gg 1$, is

$$\frac{\omega^2}{\Omega_e^2}=1+\frac{2\alpha^2}{k_\perp\rho_e}\ , \tag{13}$$

where α is the Salpeter parameter, $k_\perp$ is the fluctuation wave number perpendicular to the magnetic field direction, and ρ_e is the electron gyroradius [4]. The resonance feature is found near the first harmonic electron cyclotron frequency at 140 GHz. With the relativistic electrons, downward broadening of the resonance feature can be observed. The downward broadening effect is more dramatic with higher electron temperatures. Although not illustrated here, with very high electron temperature, the harmonics of Ω_e resonance begin to overlap due to a large broadening of the resonances.

Figure 3 shows the two branches of resonance near the electron plasma frequency (ω_{pe}) for $\phi\approx0°$ given by [4]

$$\frac{\omega^2}{\Omega_e^2}\approx1+\frac{\omega_{pe}^2}{\Omega_e^2}-\frac{\omega_{pe}^2}{\omega_{pe}^2+\Omega_e^2}\ , \tag{14}$$

$$\frac{\omega^2}{\Omega_e^2}\approx\frac{\omega_{pe}^2}{\omega_{pe}^2+\Omega_e^2}\ . \tag{15}$$

The relativistic spectrum shows shifting of both resonance features to lower frequency, with the lower frequency branch exhibiting a larger shift. At frequency offsets less than 10 GHz, the ion features, especially the slowing down α-particle feature, can be observed.

While the observance of these high frequency resonance features is not the main motivation of the current collective scattering experiments, studies of these resonances may offer additional insights to high temperature plasmas relevant to ITER and other fusion burning experiments. With proper modifications to the receiver electronics of current scattering experiments in JET and TFTR, these resonances may be observed.

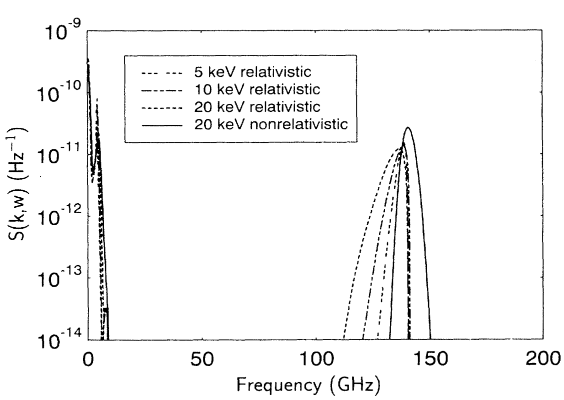

FIG. 2. Bernstein resonance at higher frequency spectrum near Ω_e. The figure compares the relativistic and nonrelativistic models. Relativistic spectra are shown for $T_e=5$, 10, and 20 keV. A nonrelativistic model is shown for $T_e=20$ keV. n_e is 1.2×10^{14} cm^{-3}, Salpeter parameter at 10 keV T_e is 3.4, θ is 6°, and ϕ is 87°.

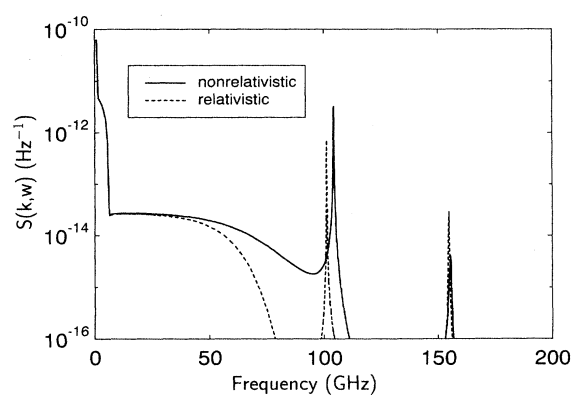

FIG. 3. Relativistic and nonrelativistic model spectra in the high shifted frequency range where k is nearly parallel to $B\,(\phi\sim0°)$. Note the ion features near the zero shifted frequency range. Electron and ion species temperatures are at 10 keV. n_e is 1.2×10^{14} cm^{-3}, the Salpeter parameter is 5.1, θ is 4°, and ϕ is 0.1°.

V. CONCLUSION

In this study we have investigated relativistic effects on the collective Thomson scattered spectra. A computer code was developed to model the electrons with the relativistic Maxwellian susceptibility tensor. It was found that minor effects of relativistic electrons can be seen in the $S(\mathbf{k},\omega)$ spectra only in scattering regimes where plasma resonances are present. However, in the application of the collective Thomson scattering to diagnose energetic ion properties, most scattering is planned to be done away from any resonance to simplify data interpretation. In these situations, relativistic effects $S(\mathbf{k},\omega)$ are negligible. Relativistic effects are more dramatic at higher fluctuation frequencies (~ 100 GHz) where the electron Bernstein resonance and electron plasma frequency resonances can be observed. Significant broadening of Bernstein resonance features toward lower frequencies and overlapping of resonance harmonic features at high electron temperatures were noted.

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